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Report on the I.U.T.A.M. Symposium on the flow of fluid-solid mixtures

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A Symposium on 'The flow of fluid-solid mixtures' was held at the University of Cambridge from 24 to 28 March 1969, under the auspices of the International Union of Theoretical and Applied Mechanics. There were 104 participants, representing 19 countries, and attendance was by invitation only. Since there will be no publication of the proceedings in full, the following condensed account of the developments described at the various sessions has been prepared for publication by three of the participants, all of whom were involved in the organization of the Symposium.

General remarks

Mixtures of a fluid and solid lumps or particles are common in various fields of engineering—hydraulic, mechanical and chemical—and of geophysics, and considerations of their motion raise many puzzling dynamical questions. The international scientific committee responsible for the planning of the Symposium (the members of which were: G. N. Abramovich, Moscow; G. K. Batchelor, Cambridge, Chairman; A. Craya, Grenoble; J. R. A. Pearson, Cambridge, Secretary; V. A. Vanoni, Pasadena) believed that it would be illuminating to try to get a total picture of the many phenomena arising in these different fields. It seemed likely that some common scientific principles would emerge; and it is always stimulating for workers in one field to hear about related ideas and observations in a neighbouring field. The participants were therefore chosen to represent a variety of fields, with different backgrounds and familiarity with different techniques, in the hope of generating an exchange of ideas and information which might otherwise not occur.

In order to give participants some idea of the intended scope of the Symposium, a number of important topics were suggested. After some adjustment of the list in the light of the prepared talks offered for presentation, the actual programme was divided into six sections under the following headings:

1. Hydraulic turbulent transport of particles.
2. Hydraulic transport: mobile beds and bed forms.

3. Pneumatic transport and particle-inertia effects.
4. Fluidization of particles.
5. Non-turbulent suspensions: motion of individual particles.
6. Rheology of suspensions.

Six well-qualified speakers were invited to present 60-minute review talks. These were intended to set the scene, in the section concerned, for the more detailed short contributions that followed each review talk. Chairmen were asked to accept a positive role in directing discussion, and were asked to summarize developments at the end of the six sections. Because of the wide-ranging nature of the Symposium, and the large number (48) of short contributions made in the week, discussions tended to reveal wide divergences of outlook and approach. The advantages gained by participants from the meeting lay therefore more in the nature of a widening of outlook than in a detailed and rigorous examination of a limited number of specific problems.

This breadth of subject-matter makes reporting of the meeting rather difficult. We have chosen here to give a section-by-section account which aims to touch on everything said at the meeting in more-or-less chronological order.

Section 1. Hydraulic turbulent transport of particles

In a review lecture on transport of solids in channels Prof. A. T. Ippen* † outlined the interesting result that in flume experiments with turbulent streams of given depth and slope the slope of the velocity profile for flow with suspended sand is greater than for clear flow. Since such velocity profiles follow the logarithmic law, an increase in the velocity gradient, with slope and depth and hence bed shear stress kept constant, means that the von Kármán universal parameter (κ) is reduced by the presence of suspended sediment. The reduction in κ also results in a reduction of the diffusion coefficient for momentum (eddy viscosity). A reduction in the diffusion coefficient for sediment as concentration is increased is also observed.

The reduction in κ and the eddy viscosity has previously been explained by postulating that since the energy to support sediment must come from the turbulence the presence of sediment will dampen the turbulence and by this means also reduce the diffusion coefficient. Observed values of κ do in fact correlate roughly with the power required to suspend observed concentrations of suspended sediment (Vanoni 1953; Einstein & Chien 1955). The hypothesis has also been advanced that the main effect of the sediment on the turbulence and other flow parameters occurs near the bed where the concentration is always the highest. Support is given to this hypothesis by the fact that the correlation between κ and the power to support that part of the suspended sediment near the bed is better than that between κ and the power to support all the sediment in suspension (Vanoni & Nomicos 1960).

† An asterisk to a name indicates that the work concerned was described in a lecture at the Symposium; details of any relevant past or proposed publication are given in the list of references at the end of the article.

Experiments by Elata & Ippen (1961) with almost neutrally buoyant particles in an open channel flow showed that κ decreased as concentration of particles increased. Furthermore, measurements with a total head tube showed that the intensity of longitudinal turbulence increased as the concentration increased. Obviously these results are in conflict with the hypothesis advanced to explain the effects of suspended sediment on flow properties. In his lecture Ippen reinterpreted and extended an unpublished theory for the distribution of suspended sediment which predicts some of the effects observed experimentally. This theory, first developed over 35 years ago when Ippen was a student of von Kármán, employs the Krey (1927) equation for the velocity profile. The resulting theory shows that κ is not a constant but a quantity depending on the concentration in such a way that the value of κ diminishes as concentration increases. It also predicts a decrease in κ for neutrally buoyant sediment as observed by Elata & Ippen (1961).

Prof. M. Hino* outlined a theory for the friction factor of a two-dimensional flow with suspended particles based on his earlier publications (1963*a*, *b*, 1965). The flow is divided into two regions, the inner 'law of the wall' region and the outer 'law of the wake' region. The total rate of energy dissipation in these two regions, taking into account the increase in effective viscosity due to the suspended particles and the energy to suspend particles, is equated to the total energy production. This results finally in an equation for the friction factor in terms of mean particle concentration, fall velocity and specific weight of the particles, the effective viscosity of the fluid in the presence of particles and the parameters describing the structure of the turbulent flow. The theory indicates that κ decreases with increase in concentration of particles regardless of their specific weight and that turbulence intensity is increased by the addition of neutrally buoyant particles.

The theoretical results agree with observations in smooth flumes (Elata & Ippen 1961), and a pipe (Daily & Chu 1961; Daily & Hardison 1964) with neutrally buoyant particles in which the friction factor increased with increasing particle concentration. They also agree with results of flume experiments with sands in water (Vanoni 1964; Vanoni & Nomicos 1960), in which the boundary was fixed, completely rough and had no movable bed forms on it. In the latter flume experiments the friction factor decreased as concentration increased. Hino's theory indicates that channels with rough walls show a reduction in friction factor with increase in concentration even for neutrally buoyant particles and that smooth channels show a decrease in friction factor only with particles denser than the fluid. Unfortunately there are no data for rough channels with neutrally buoyant particles and for smooth channels with heavy particles to test the theory further.

The result that the addition of neutrally buoyant particles to a flow causes the turbulent intensity to increase and the von Kármán constant and hence the eddy viscosity to decrease is difficult to understand intuitively. Further studies are needed to clarify this result, particularly in the measurement of turbulence in flows containing particles. Some measurements by Daily & Hardison (1964) indicate that for flows with high concentrations of neutrally buoyant particles

(10% by volume or more) in pipes, the intensity of longitudinal turbulence has its minimum value near the wall where in clear water flow it reaches a maximum. Near the wall the measured turbulence intensity with suspended particles is only slightly greater than that with clear water but at the centre it is almost three times that for clear water.

Brig. R. A. Bagnold* adopted the interesting hypothesis that the turbulent velocity fluctuations normal to a wall are more intense in the direction away from the wall than toward it. He reasoned, although the argument was disputed, that the equilibrium of suspended particles demands that this type of asymmetry exist in the turbulence velocities normal to the bed of a stream.

He made two sets of experiments to test his hypothesis. In the first set a rimless wheel with spokes was mounted over a narrow flume with its axis horizontal and normal to the flow direction and with the spokes immersed one-third their length into the flow. Measurement of the torque on the wheel about an axis parallel to the flow indicated that there was a force on the spokes outward from the wall. The second set of experiments was made with a cylindrical static pressure tube, similar to a Pitot-static tube without the total head opening, but with only a single port in it. The tube was made so the portion with the pressure port in it could be rotated while in place thus making it possible to read the static pressure, first with the port facing the bed of a wide channel and then facing upward toward the free surface. The torque on the wheel and the difference in static pressures on the lower and upper sides of the tube both indicated that the pressure acting away from the boundary exceeded that toward the boundary. There is considerable question regarding the meaning and interpretation of the measurements. For example, it is possible that the force on the spokes of the wheel results from lift due to the velocity gradient and that the unknown response characteristics of the static pressure tube yield spurious results.

The standard theoretical expression for the distribution of suspended sediment (Rouse 1937) over the depth of a steady uniform turbulent stream contains the fall velocity of the sediment particle and the turbulent diffusion coefficient for sediment. The fall velocity is usually assumed equal to that of the particle in the still fluid and the diffusion coefficient is assumed equal or proportional to the eddy viscosity (the Reynolds analogy). Prof. W. M. Sayre* reported some novel experiments in a large flume made jointly with Jobson (see also Jobson 1968) in order to determine fall velocities of sediments and diffusion coefficients for sediment and dye in turbulent flows. The results of this study are: (1) the fall velocity of sediment particles is larger in turbulent flow than in still water, with the difference tending to be more pronounced for fine glass spheres (0·123 mm) than for medium-sized sand particles (0·390 mm); (2) the magnitude and distribution of the vertical turbulent diffusion coefficients for dye and glass spheres are close to that for momentum transfer thus supporting the Reynolds' analogy; (3) the diffusion coefficient for the medium sand was somewhat smaller than that for the glass spheres and more heavily weighed toward the bed; (4) secondary flows were present in the flume but were shown to have only minor effect on the main results.

In the experiments a continuous stream of dye or of particles was injected uniformly across the width of the flume at the water surface. The vertical distributions of the dye or particles was determined by sampling at several sections downstream from the injection point. Local values for the diffusion coefficients and the fall velocity of the particles were determined by analyzing the data by applying the conservation equation for the dispersant in a thin prism of the flow bounded by two cross-sectional planes, the water surface and a plane parallel to the surface. The authors hypothesize that the rotation of the fluid causes particles to be spread outward from the centre of rotation, thus adding to the particle diffusion. The fact that the diffusion coefficient for particles tends to be weighed towards the bed where the rotation is highest is taken as evidence that this mechanism is at work.

Prof. L. M. Brush* described experiments by himself, Fox and Ho to investigate the effect of turbulence on particle fall velocity. They measured fall velocities of spherical particles settling in water in a container that was oscillated horizontally or vertically. For fine particles in the Stokes range the mean fall velocity was equal to that in still water. Larger particles showed a systematically lower fall velocity in the oscillating system which reached a value of as little as 50 % of the free fall velocity for accelerations believed typical of natural channels. When fall-velocity corrections are made, based on the above experimental results, to results obtained in isotropic turbulence fields the authors found that the diffusion coefficients for sediment and momentum were equal. For shear flows the results were contradictory, with the ratio of the coefficients being sometimes greater and other times less than unity. A few experiments suggested that with a flat sediment bed the two coefficients have the same value.

The findings of Sayre and Jobson and of Brush *et al.* on fall velocity are clearly in conflict. Brush *et al.* report a decrease in fall velocity in the oscillating water container with the maximum decrease occurring with the coarse particles. Sayre and Jobson, on the other hand, found that the fall velocity was higher in turbulent flow than in still fluid and that this effect was more pronounced for the finer sediment. The observations on diffusion coefficients for sediment were only partly in conflict. One obvious conclusion is that the shaking box used by Brush *et al.* is not a good model for turbulence. However, this deduction may be premature and it seems safer to defer the conclusion until further studies can be made.

Prof. J. F. Kennedy* reported results of experiments on turbulent flow of a suspension of neutrally buoyant particles in a two-inch pipe based on a study by Roberts, Kennedy & Ippen (1967). The friction factors for the suspension were slightly lower than those of clear fluids at the higher Reynolds numbers and slightly higher at lower Reynolds numbers. Friction factors calculated on the basis of the mean measured fluid velocities collapsed to a single curve for all concentrations and particle sizes. Velocity profiles inferred from measured friction factors indicated that the velocity profiles of suspensions are sharper than those for homogeneous fluids, indicating a decrease in the von Kármán constant, as previously reported (e.g. by Elata & Ippen 1961). The results also showed that the average velocity of the particles exceeds that of the liquid, possibly because a particle is excluded from the slow-moving layer near the wall, as in the work of Batchelor, Binnie & Phillips (1955). The longitudinal dispersion coefficient of the liquid phase increased with particle concentration, a consequence of the sharpening of the velocity profiles. The dispersion coefficient of the particles also increased with concentration as a combined result of the sharpening of the velocity profiles and the inhibited lateral mobility of the particles.

Dr A. C. Bonapace* presented an analysis of the flow of a fluid-solid mixture in a pipe in which the particles are denser than the fluid. The objective of the analysis was to predict the condition for which the friction factor reaches the value for clear fluid when presumably there are no deposits of sediment on the invert of the pipe. The analysis involved considerations of the energy required to transport the sediment and a questionable assumption regarding its dependence on the difference in mean velocity of particles and fluid. Some numerical calculations were presented but no comparisons were made between observation and the author's theory.

In flume studies of the movement of coarse sediment particles Dr I. K. Hill* noted that there was a sharp demarcation between an outer region with fully developed turbulence in which very little particle interaction occurred and a region close to the bed where particle interaction was the predominant mechanism for transferring momentum in the transverse and longitudinal directions (see Hill 1967). He also observed that the boundary between these two regions appeared to occur at about the same particle concentration in the range of Shields dimensionless shear stress (defined in several works, e.g. Henderson 1966, p. 413) of 0·2–1·0 and that the volume concentration in the lowest part of the layer was about 0·54. With the aid of this information and dimensional analysis of some ideas on intergranular stress advanced by Bagnold (1954) he developed expressions for sediment discharge in the lower region and velocity at the upper edge of the lower region. These expressions contained integrals that were functions of the particle concentrations at the upper and lower edges of the lower region and are constant if these concentrations are indeed constant. It has not yet been possible to compare the theory with observations.

Prof. G. F. Round* described research in capsule transportation by liquids in pipelines where the capsules or units to be transported have diameters as large as about 0·9 times the pipe diameter and lengths up to 5 or 6 pipe diameters. The critical velocity at which such capsules become suspended is much less than for particles of the usual size and the power required to transport the capsules is much less than for particulate materials. Small and large scale laboratory experiments on capsules have been carried out and a feasibility study was undertaken to appraise the method and delineate the problems to be solved in order to make it technologically operative. Results of experiments with capsules in a $1\frac{1}{4}$ in. pipe have been reported by Round & Bolt (1965). The mechanics of a capsule in a turbulent flow is still to be worked out.

Prof. G. V. Middleton* showed a fascinating motion picture of turbidity currents, that is, sediment-laden flows driven by density differences, in a flume 5 cm wide by 5 m long. Turbidity currents bearing suspended sediment as coarse as sands are important because they are thought to be responsible for depositing sandstones of a certain type which show a decrease in grain size from bottom to top of the deposit. Geologists are presently interested in developing criteria for distinguishing sand beds deposited by turbidity currents from those deposited by ocean currents or other mechanisms. The mechanism for laying down sediment beds is not well understood. The motion picture which showed beds being formed by turbidity currents was shown in an attempt to interest workers in fluid mechanics in studying this important phenomenon.

Section 2. Hydraulic transport—mobile beds and bed forms

The second section dealt mainly with the interaction of the flow and the bed of an alluvial stream and the prediction of the sediment discharge rate. The subject of mobile bed forms is a central and imperfectly understood one in the mechanics of alluvial streams. Its clarification is necessary to the solution of such key problems as the determination of the relation between the velocity, depth and discharge of streams and their capacity to transport sediment.

The survey lecture was given by Dr A. J. Reynolds,* who discussed the processes involved according to their time scale. The time-spans of interest to geophysicists who study the development of land forms and entire river systems are measured in eons. Phenomena such as the migration of river meanders and occurrence of capital floods which are of interest to the engineer occur in intervals of a few decades. Those interested in research on river mechanics, on the other hand, are interested in phenomena with short time-scales. Among these are migration of dunes which take place in a matter of days or hours, and turbulence, other pulsations in flow velocities and fluctuations in the forces exerted on the sediment which occur in fractions of a second. The author illustrated his presentation with diagrams of river systems on a continental scale, aerial photographs of rivers, photographs of bed forms in flumes, and graphs showing results of the analysis of the formation of dunes, ripples and other bed forms.

Prof. V. A. Vanoni* presented a short two-sequence motion picture showing the intermittency of sediment movement and fluid velocity at the bed of a flow with a growing turbulent boundary layer in a flume 39 cm wide by 4 m long. The first sequence (described in Vanoni 1964), showed the highly unsteady motion of grains of 0·1 mm quartz sand in a portion of the bed about 1 × 1 cm in size when threshold conditions for initiation of motion obtained. The Reynolds number formed by the product of shear velocity and grain diameter was about unity, indicating that the grains were well submerged in the laminar sublayer. In the second sequence (described in Sutherland 1967) the fluid motion near the bed was made visible by a stream of dye allowed to seep up through a bed of 0·56 mm quartz sand during flow conditions near the threshold value for starting motion of the sand. Most of the time the stream of dye on the bed was laminar but at random intervals the dye was seen to erupt into the flow for distances as large as several centimetres. This eruption has been observed to be associated with the entrainment of particles (Sutherland 1967).

Dr A. J. Raudkivi* presented results of an interesting study by Apperly and himself based on a thesis by Apperly (1968) in which the three components of force on a $\frac{1}{4}$ in. diameter particle in a flat bed of similar particles were measured in a turbulent flow about 9 in. deep. When the grain was embedded with the others the mean values of the drag and lift forces were approximately the same and were about equal to the product of the bed shear stress and the projected area of the grain. As the instrumented particle was raised above its neighbours the lift and drag increased. When the particle was $\frac{1}{4}$ times its diameter above the others the drag and lift indicated the kind of unsteadiness illustrated in the pictures shown by Vanoni.* The spectrum of the drag showed a similarity to that for the longitudinal components of turbulence.

Dr C. F. Nordin* presented a derivation of dimensionless frequency and wave-number spectra for sand ripples and dunes which occur on beds of alluvial streams subject to the condition that mean values of velocity and sediment discharge do not change with time or with position along the stream. The forms of the spectra were derived from river and flume data covering a range of flow depths from about 3 cm to 1 m. The variance of the bed-profile records of ripples and dunes was found to correlate well with the square of the flow depth so the effect of depth is partially accounted for by standardizing the records to zero mean and unit variance before making spectral analyses. For the limited conditions considered, peak ordinate values of the spectra are well correlated with flow velocity. At the higher wave-numbers and frequencies, the dimensionless spectra follow the $(-\frac{1}{3})$-power of wave-number and the (-2)-power of frequency, implying that wave celerity varies as the square root of wave-number. In the range of higher frequencies and wave-numbers, the shapes of the spectra derived from the data agree, at least in part, with the shapes of the theoretical spectra. This report represents a continuation of work reported previously (Nordin 1966, 1968).

Prof. F. Engelund* outlined a theory for the development of antidunes which differs from those of Kennedy (1963) and Reynolds (1965) by taking into account the non-uniform distribution of the suspended sediment and the dynamic consequences of the resulting density gradients. The motion of the fluid-solid mixture is described by a vorticity transport equation and the vertical distribution of suspended sediment is expressed by the unsteady diffusion equation. These two equations are linearized and are then assumed to be perturbed by a small periodic component and thus result in two coupled ordinary differential equations. The sediment concentration at the bed which appears in the boundary conditions is expressed as an empirical function of hydraulic parameters. A stability analysis of these equations finally gives the condition under which perturbation of the bed will increase or attenuate and thus delineates the conditions under which antidunes will form. The results for the variation of Froude number with wave-number show better agreement with observations than those of Kennedy (1963) and Reynolds (1965).

Prof. T. Blench* argued, with the help of dimensional analysis, that flow in a flume with an erodible bed of granular sediment can be described by one equation for the friction factor and another for the Froude number. Each of these quantities is expressed as a function of six dimensionless variables of the system. In support of his assertion he cited the work of Cooper & Peterson (1968) in which world flume data correlated reasonably well according to these relations and displayed the effect of bed form on the relations. He stated that the large discrepancies between the many available empirical sediment discharge formulas result because they are based on curves fitted to different samples of data which are not representative of all conditions. He also stated that the formulas of régime theory for friction factor (Blench 1966) contain the proper variables and appear to be general in nature; a statement on which there is not complete agreement (e.g. see Engelund 1967). The data on alluvial flows were found to be inconsistent and sometimes incomplete and a plea for minimum standards in laboratory work was voiced.

Experiments in a flume with a bed of fine sand have shown that for certain pairs of values of flow depth and stream slope there are two sets of values of mean velocity, water discharge and sediment discharge that will yield uniform steady flow conditions (Brooks 1958). In one case the bed is covered with ripples, is very rough and the velocity and sediment discharge are low. In the other the bed is flat and the velocity and sediment discharge are high. This phenomenon explains the discontinuity in curves of discharge against water surface elevation at gauging sections of some sand bed rivers (Colby 1960).

The above relation leads to the statement that the velocity, water discharge or sediment discharge of an alluvial stream cannot be expressed as unique functions of depth and slope or of the bed shear stress. Dr T. Maddock* showed that this indeterminacy can be avoided by expressing the relations between the variables as follows: (1) sediment discharge and slope are functions of water discharge and depth; (2) water discharge, velocity and slope are functions of sediment discharge and depth; (3) velocity, depth and slope are functions of water and sediment discharge. Maddock* noted that self-formed channels operate according to statement (3) and that they do not always behave like flumes, which often operate according to statement (2). He also called attention to the empirical relation for self-formed streams that velocity is directly proportional to the square root of the sediment size (Maddock 1968). He further observed that the empirical velocity relations of Lacey (1958), Inglis (1968), Engelund (1966), Blench (1966) and Einstein & Barbarossa (1954) are of this form.

Measurements of the sediment discharge of rivers is of importance not only in solving specific problems but also in improving relations for predicting this quantity. Reliable samplers are available for measuring the suspended sediment discharge but bed load samplers are inaccurate and better methods are needed for measuring the bed load discharge. One method that may show improvement over the conventional bed load sampler involves the use of tracers.

Dr M. de Vries* described experiments in a flume 3 m wide in which sand grains dyed with fluorescent dye were used as tracers to measure the bed load discharge of a sediment composed of sand and gravel. The theory used to determine the sediment discharges from the observations (presented in de Vries 1966), describes the dispersion of the tracers by a Fickian diffusion model in which the various grain sizes are considered separately. The theory does not require that the grain size distribution of tracer sediment be the same as that of the bed sediment. The parameters required to describe the grain motion are determined from the measurements which consist mainly of bed sediment samples. The technique of measurement with tracers is still in its developing stage and is far from ready for routine field use.

Mr R. Fernández Luque* outlined a theory for erosion of a sand bed (Luque 1967), which takes into account the seepage flow through the sand and the resulting forces on the grains near the bed surface. This theory gives the forces due to seepage flow in the vicinity of a region of increased transient pressure which tend to dislodge the grains. The intense erosion in the stagnation zone of the upstream face of a ripple is explained in terms of this mechanism. The observed crater in the bed of a submerged horizontal bed of sand formed by a vertical water jet is also explained in this way. Others who have studied erosion by jets, e.g. Rouse (1940), and Laursen (1952), and initiation of motion on a rippled bed, e.g. Sutherland (1967), interpret their results in terms of the drag and lift on particles due to the stream flow. From their observations these workers found no evidence that the seepage forces were significant. This same point of view was expressed in the discussions of the paper by Luque.

Dr O. F. Vasiliev* described a set of partial-differential equations which he derived for the case of non-uniform flow of a sediment-laden stream in which the bed elevation varies slowly with time due to the deposition or erosion of sediment. The relations were based on the continuity equation for water and sediment, the momentum equation and the Levi (1957) equation which expresses sediment discharge as an empirical function of flow velocity. The velocity was based on the Manning formula in which the resistance coefficient was assumed proportional to the $\frac{1}{6}$-power of the sediment size. The relations are an extension of some relations presented previously (Vasiliev 1958). The equations were integrated numerically by simulation for the case of flume studies carried out by Rusinov & Zadvorny (1956) in which a stream carrying uniform sediment was allowed to aggrade. The calculated profiles of the sediment surface agree well with the observed ones. This is remarkable in view of the fact that empirical sediment discharge equations tend to be very inaccurate and also that the assumption of constant friction factor is not usually a good one for alluvial streams. Further development is needed before the good agreement between calculations and observations obtained in this laboratory case can be expected in the much more complicated cases of natural streams.

Prof. P. Peter* outlined a method for predicting the stability of soil embankments against piping under elevated ground-water pressures such as occur at the exposed faces of some earth dams and dikes. This method expresses stability of non-cohesive material in terms of the stabilizing force of particle weight and the destabilizing fluid forces. The analysis also takes care of cases where cohesive forces are acting and takes account of the effect of grain size and shape. Some aspects of the analysis used by Housner (1958) in studying sand blows in earthquakes are applicable to the piping problem.

In an interesting talk on modern dredging practice Ir. J. de Koning* brought out the importance of turbidity currents to this field. For instance he reported that material dredged from borrow pits flows from the caving banks of the pit to the dredge intake as a turbidity current. By putting the dredge intake as deep as 70 m a crater 400 m in diameter has been formed. This reduces costs not only because the dredge is moved only infrequently but also because when the material is fed to the intake by turbidity currents a very high concentration (as much as 50 % by volume) can be maintained in the discharge line. A high-velocity water-jet at the intake was also found to increase the sediment concentration in the discharge line. Detrimental effects of turbidity currents occur in placing dredged material. When material is loaded on a barge these currents may cause a large fraction of the sediment to flow overboard and be wasted. Also, in placing fill, either directly from the dredge line or by dumping from a barge, turbidity flows tend to carry sediment away from the embankment into deep water where it is lost. de Koning* also brought out the economic importance of efficient handling of dredge materials and indicated that advances in the mechanics of turbidity currents and other sediment laden flows will contribute to improvements in dredging.

Section 3. Pneumatic transport and particle inertia effects

When the continuous phase is gaseous rather than liquid, the ratio of particle density to fluid density is generally large, and this affects the flow in two ways. First, the particles have much greater inertia than the displaced fluid, and in an unsteady or accelerating flow the particle paths may therefore differ very significantly from the paths of fluid elements; solid particles will be less easily accelerated than fluid elements, and their paths will on average be less curved, an effect of particular potential importance in turbulent flows. Secondly, a significant fraction of the available transfer of energy from mean flow to turbulence may be required to maintain the particles in suspension against gravity; this leads to a drain of energy from the turbulence, and perhaps to a damping of turbulence in regions where the particle density is high.

Section 3 of the symposium began with a survey lecture by Prof. P. R. Owen*, the text of which is being printed in full in this issue of the *Journal* (see page 407). He described, in a most lucid manner, a complete classification of the many important physical effects in the subsonic regime in terms of ratios of time-scales for each of the physical processes involved. In so doing he mentioned such widely varied aspects as Brownian motion, electrostatic force effects, dune formation, and saltation.

Owen touched on the problem of particle motion in a turbulent air jet at the end of his lecture, and this was followed by an account from Prof. V. W. Goldschmidt* of measurements of concentration profiles of particles injected into such a jet flow. The results were expressed in terms of a turbulent Schmidt number, the ratio of eddy viscosity to eddy diffusivity for the particles. The particle size range was 1–100 μm, and the corresponding values of the Schmidt number ranged from 1 to 6, which is consistent with the notion that the more inert particles are less easily diffused by the turbulence.

A further paper on the effect of particles in a jet flow had been submitted to the Symposium, but was not finally delivered, as the author, Prof. G. N. Abramovich, was unable to attend. According to the abstract of the paper, the theory is concerned with the damping of turbulent fluctuations due to the transfer of energy from the fluid to the particulate phase; this leads to a decreased spread and an increase in the penetrating power of the jet. The theory differs in certain respects from that outlined by Owen, who supposed that the particles were fully responsive to the turbulent fluctuations and assumed that their main effect was to cause a modification of the mean velocity profile due to particle migration across the mean shear.

Three papers in the section were concerned with supersonic flow phenomena. In the first Prof. W. Wuest* described extensive calculations on the hypersonic flow of dusty gases round two-dimensional bodies. The method followed the familiar precedure for particle motion in potential flow, but in this case the flow was either supersonic wedge flow, inviscid stagnation point flow, or viscous hypersonic free molecule flow around a plate. Moreover, the resistance law was either the Stokes law, or the square law for small particles in hypersonic flow, so a very comprehensive set of results is given. One wondered what happens when a particle passes through an oblique shock wave; a straight particle path was assumed, but in fact the particle must suffer a lateral impulse and this may not always be negligible.

Prof. P. P. Wegener* described very comprehensive experiments on the flow of moist air through a convergent-divergent nozzle, the very small condensed moisture particles being observed by Rayleigh scattering of laser light. Satisfactory agreement with the assumption of homogeneous nucleation was obtained, the droplets being very small and numerous with counts of order $10^{12}/\text{cm}^3$.

A theoretical discussion of the conditions for the existence and uniqueness of steady compression waves in dust-laden gases was given by Dr B. Schmitt-von Schubert*. An interesting feature of the results is that a *continuous* steady compression wave is possible when the upstream gas speed lies between the equilibrium speed of sound and the frozen speed of sound, dissipation in the transition region being provided by the relative motion of gas and particles.

The last paper was an account (and film) by Ing. P. S. Kucera* of an interesting device designed to reduce erosion in pneumatic flow round a pipe bend, and consisting of a screw placed upstream so that the water entering the bend swirled about the pipe axis. In this way, the particles were centrifuged towards the pipe wall along which there was thus a tangential particle motion and reduced erosion. One of the questions that was raised was: might it be preferable to have two screws upstream, so placed as to reinforce the secondary flow that already exists in the pipe bend? Although the author did not say so, the film showing rapid erosion in a simple 90° bend appeared to indicate that erosion was due to a focusing effect whereby particles struck a very small region of the inner surface of the pipe bend.

Section 4. Fluidization of particles

The section began with a survey lecture by Prof. K. Rietema,* the gist of which was to put what might be described as a 'non-establishment' viewpoint about the basic theories of fluidization. Although the writer (J. F. D.) did not agree with all that was said, it is always stimulating to have a lecture which attacks what have come to be regarded by many workers as well entrenched theories. Rietema's main hypothesis was that the rise of bubbles in a fluidized bed is best described, not by potential flow theory, but by a mechanical stress theory of the type familiar to workers in soil mechanics; the rise of a bubble would thus be governed by continuous failure of arches above the bubble. The objections to the potential flow theory were set out as follows.

(1) Measurements for bubbling fluidized beds show a quite high electrical conductivity, indicating that particles are continuously in contact, though the potential flow theory proposed that particles are virtually out of contact.

(2) Bubble shape and size, and to some extent velocity, depend upon the nature of the particles.

(3) The pressure distribution predicted round the rear of the bubble, by potential flow theory, is not consistent with the requirement of constant pressure within the bubble.

(4) Measured drift profiles are not too similar to the prediction of potential flow theory, nor is the mode of bubble coalescence the same as for gas–liquid systems.

(5) The fact of observed particle slip at the wall is said to invalidate the analogy between a fluidized bed and a liquid.

Against these objections, the following may be said:

(1) Very light contacts between particles would surely be enough to give measurable electrical conductivity without inducing sufficient shear stress to invalidate the potential theory.

(2) Many thousands of bubble-velocity measurements in fluidized beds for all sorts of particles, and a wide range of bubble sizes, agree remarkably well with the results for bubbles rising in low-viscosity liquids, i.e. $U_b = 0{\cdot}71(gD_e)^{\frac{1}{2}}$ for a bubble of diameter D_e in a large container, and $U_b = 0{\cdot}35(gD)^{\frac{1}{2}}$ for a bubble (slug) in a tube of diameter D; here U_b is the bubble velocity.

These two results for U_b have a sound theoretical basis (Davies & Taylor 1950), and figure 1 shows how they compare with the available data for gas bubbles rising in liquids. Figure 2 shows the corresponding results for air bubbles rising in air-fluidized beds. The data exhibit a good deal of scatter, but there is general agreement with the formulae quoted above for gas bubbles in liquids. In considering the scatter in figure 2, it should be noted that there is a similar proportion of scatter for the results given by Davies & Taylor (1950) and plotted in figure 1; the scatter seems to be characteristic of large bubbles in liquids, and it may be that such bubbles do not rise steadily, and may thus not have an accurately defined rising velocity. This could also be true of bubbles in fluidized beds. On the other hand, the scatter of data in figure 2 may be due to interparticle shear stresses of the kind discussed by Rietema. But it is clear that to a first approximation the fluidized bed data agree with the results for low viscosity liquids.

The main references describing bubble velocity measurements in fluidized beds are given in the legend to figure 2. A recent summary of bubble velocity measurements is also given by Kunii & Levenspiel (1969).

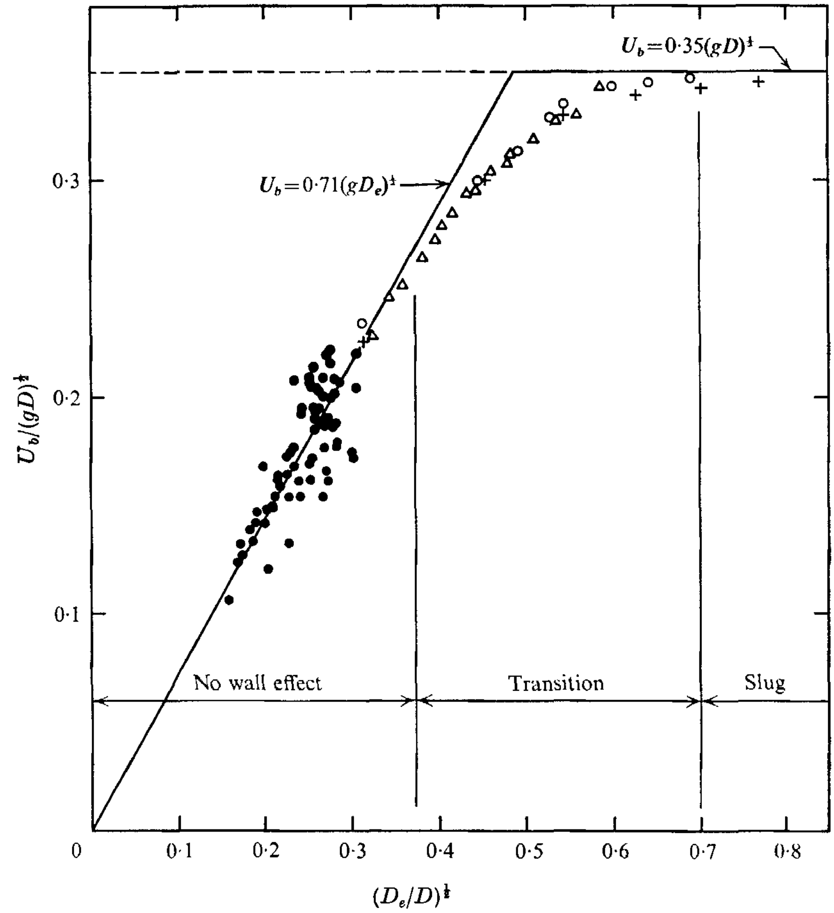

FIGURE 1. Bubble rising velocity in water. Wall effect. △, Uno & Kinter (1956): $D = 0\cdot069$, $0\cdot095$ and $0\cdot152$ m. +, Collins (1967): $D = 0\cdot092$, $0\cdot193$ and $0\cdot295$ m. ○, Calderbank (1967): $D = 0\cdot101$ m. ●, Davies & Taylor (1950): $D = 0\cdot762$ m.

Extensive measurements on elongated bubbles or 'slugs' in fluidized beds are described by Ormiston, Mitchell & Davidson (1965) and by Kehoe (1969). These data are for a wide variety of materials and tube sizes and in general the slug velocity agrees well with the result $U_b = 0\cdot35(gD)^{\frac{1}{2}}$, though in certain cases Kehoe (1969) observed higher velocities because the slugs tended to move up the wall; in this latter case, following a conjecture of Birkhoff & Carter (1957) the slugs tend to rise as if they were in a tube of diameter about $2D$, and Kehoe found agreement with the above formula but with $2D$ in place of D. This

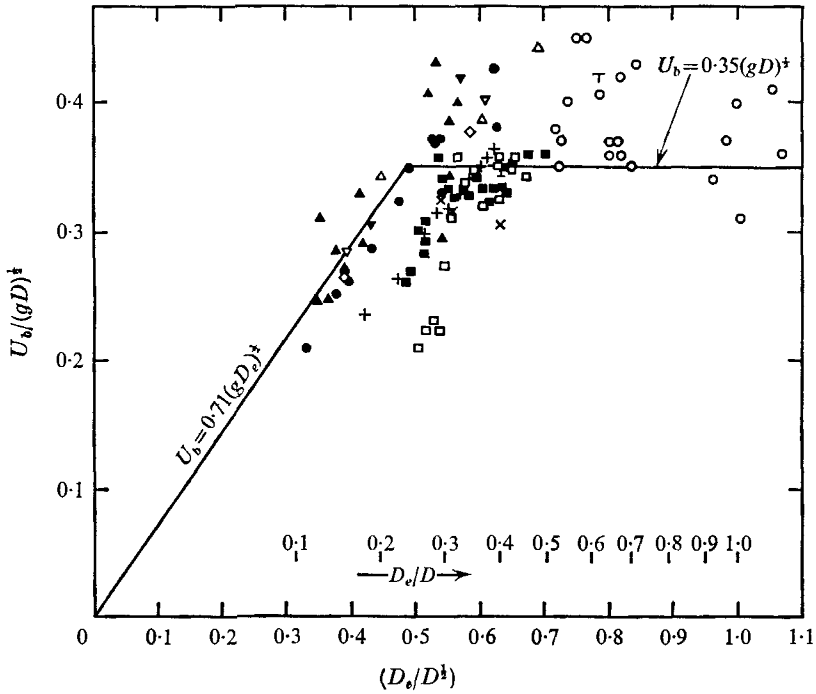

FIGURE 2. Wall effect on bubbles in fluidized beds.

	Particles	Diameter (μm)	
+	Sand	175	Davidson & Harrison (1963), p. 34
×	Glass beads	150	
⊥	Sand	400	
T	Seeds	1700	
●	Ballotini	60–550	Rowe & Partridge (1965)
▲	Silver sand	72–500	
▼	Acrylic granules	121	
◇	Synclyst	52	
▽	Magnesite	240	
△	Crushed coal	410	
□	Ballotini (A)	230	Angelino *et al.* (1964)
■	Ballotini (C)	230	
○	Coke	344	Park *et al.* (1969)

question is also discussed by Stewart & Davidson (1967) who describe further experimental evidence that slugs in fluidized beds behave as if they were in a low-viscosity liquid. Additional evidence for the validity of the result

$$U_b = 0{\cdot}35(gD)^{\frac{1}{2}}$$

as applied to fluidized beds, was given by Matsen in his contribution to the Symposium (see below).

(3) The inconsistency of pressure round the lower part of the bubble is an objection that applies equally when inviscid flow theory is used for ordinary liquids or for fluidized beds. Inviscid flow theory—interpreted literally—

predicts no drag on a bubble in an ordinary liquid; but the drag is due to the pressure defect in the wake caused by eddying, and this explanation can apply either to a bubble in a fluidized bed or in an ordinary liquid. The conclusion is that the potential flow theory is valid *only* for the upper part of the bubble—and this was all that was ever really claimed for it—though some authors seem to have assumed that potential flow theory describes the complete motion.

The variety of bubble shapes observed in a fluidized bed can certainly *not* be explained by the potential flow theory. But the different shapes may be due to bed viscosity effects, and it should be noted that the bed viscosity may alter the shape considerably, and simultaneously have only a second-order effect on rising velocity. Interparticle shear stress may also affect the bubble shape. But it is hard to see the whole mechanism of the bubble's rise being governed by interparticle shear stresses— the arch type theory—without making some allowance for the mass-acceleration of the particles. And if the interparticle shear stresses due to cohesion are small, as for a wide variety of particles, and particle inertia is allowed for, it is difficult to escape the conclusion that potential flow theory is a good approximate way of describing the motion. Certainly no theory based exclusively on soil mechanics would give a *rate* of bubble rise. The theory of Coulomb friction and plastic yield tells only whether or not a building will fall down, not how fast it will fall!

In the writer's view, potential flow theory gives a good first approximation for describing particle motion around the nose of a bubble in a fluidized bed. But the theory should be applied with caution—as with any problem of real fluid motion. It is not so long since hydraulic engineers regarded all potential flow theory as irrelevant to reality; it is to be hoped that chemical engineers will not resurrect this out-of-date attitude.

(4) With regard to drift profiles and coalescence, the similarity between bubbles in ordinary liquids and in fluidized beds is open to question. The drift profile caused by an air bubble in water is not even vaguely similar to the prediction of potential flow theory—because of the wake effect— but this does not invalidate the potential flow theory for describing the fluid motion round the bubble cap (Davies & Taylor 1950). In fact the drift profiles for a bubble in a fluidized bed agree with potential flow theory a great deal better than do the corresponding measurements for air–water! As for coalescence, this has been extensively studied for fluidized beds (see Harrison & Leung 1962; Toei & Matsuno 1967), and the results can be interpreted in terms of the analogy with gas–liquid systems.

(5) The fact of observed wall slip in fluidized beds vitiates the comparison with real liquids, but not with potential flow theory in which wall slip is possible. In this respect the fluidized bed is more 'ideal' than a real liquid!

Prof. O. Molerus* described his theory to predict the minimum bubble size in a fluidized bed. The basis of his argument was to compare the expansion of bubbling and non-bubbling beds; he argued that if a bubbling bed is to be stable, its expansion must be less than that of a non-bubbling homogeneous bed operating at the same velocity. It follows that a bed with very small bubbles is unstable, because the small bubbles rise very slowly, giving a long residence time for the fluid and hence greater expansion than with particulate (isotropic) fluidization. The theory thus predicts a minimum bubble size which is much larger for gas-fluidized beds than for liquid-fluidized beds, and this agrees qualitatively with observations. Molerus's theory complements earlier theory (Davidson & Harrison 1963, p. 80) to predict the maximum bubble size, which is larger for gas-fluidized than for liquid-fluidized beds. We are thus led to the conclusion that there is a finite range of bubble sizes which can exist; for an atmospheric air-fluidized bed, the size range would be typically from a few millimetres up to perhaps a metre; for a water-fluidized bed of glass beads, the bubble size range would be from a few microns to a few millimetres. These predictions are roughly in accordance with observations, but precise verification is very difficult.

Prof. L. Massimilla's* photographs showed the small-scale arrangement of particles over an area 6·8 × 4·5 mm, within a bed of fine particles of diameter about 60 μm. Such particles do not exhibit bubbling until the fluidizing velocity is somewhat greater than the value for incipient fluidization. The photographs showed fixed cavities of size equal to a few particle diameters and it appears to be the development of such cavities that causes the bed expansion between incipient fluidization and visible bubbling. Godard & Richardson (1968) studied the expansion of similar particles in the same range of flow rates. They correlated results in the form $U \propto \epsilon^n$, where ϵ is the mean voidage fraction, and U is the superficial fluidizing velocity; the empirical index n was found to be somewhat higher than for ordinary particulate fluidization. The higher values of n may be due to the fixed cavities observed by Massimilla, since they would afford an easy passage for the fluid to pass through the bed. The formation and breakdown of such cavities is probably governed by Rietema's theories, while the moving bubbles which predominate at higher velocities are described by the potential flow theory.

Dr R. Matsuno* described experiments to measure the interphase transfer rate between a rising bubble and the particulate phase by injecting a bubble containing CO_2, and then sucking it out further up the bed. This is the sort of experiment one dreams about, and then dismisses as impracticable, but Toei & Matsuno have done it, and give what appear to be very consistent data. Their measurements of a fluctuating bubble velocity are less plausible, and this casts doubt on the validity of their interphase transfer theory, but there is no doubt as to the value of the experimental work.

Dr K. Østergaard* described the well-known paradox for liquid-fluidized beds which contract—instead of expanding as might be expected—when a gas flow is added to the liquid flow. This apparent anomaly can be explained by supposing that the air bubbles carry liquid with them, thus forming composite gas–liquid bubbles. The composite bubbles rise according to the equations plotted in figure 2 but no theory has yet been formulated to predict their maximum size; it is clear, however, that the presence of the gas means that much larger bubbles can be formed than with liquid alone. Thus the composite bubbles are much larger than would be predicted by Molerus's theory with the liquid fluidizing the particles.

The work of Prof. E. A. Nepomniaschchy* dealt with a general equation based on random-walk ideas, but which many of us would recognize as a diffusion/convection equation, which was solved with specified boundary conditions. This was certainly the T. part of the I.U.T.A.M. Conference, and one felt the need for a link with reality; how would this theory be applied, for example, to the problem of vibrating screens (applied mathematicians will know what these are, just as engineers know about ordinary Markovian processes)?

Dr J. M. Matsen described his theoretical work on the expansion of fluidized beds in tubes. This was the kind of work we all hope for, a simple theory which explains a wide variety of published data and removes apparent paradoxes that have puzzled previous investigators. Matsen's idea is quite simple: when a fluidized bed operates at high gas velocity, the large bubbles or 'slugs' cause the surface to move up and down with an appreciable amplitude. Observers have noted the maximum and minimum bed heights but were unclear as to whether minimum, maximum, or mean height should be used in comparisons with theory. Matsen showed clearly that it is the maximum height which is relevant. His theoretical argument depends upon what happens when the bubble flow is started, and this leads to the result

$$\frac{H_m - H_0}{H_0} = \frac{U - U_0}{0{\cdot}35(gD)^{\frac{1}{2}}}, \tag{1}$$

where H_m is the maximum bed height, U is the superficial gas velocity, and H_0 and U_0 are the height and velocity at incipient fluidization. Matsen showed that (1) agrees with a very wide range of published data, some of it 21 years old. His work is convincing additional proof that bubbles in a fluidized bed behave much as they would in a low viscosity liquid, and that this is true not just for single injected bubbles, but for bubbles in a bed operating at a very high fluidizing velocity. This—together with the fact that the work was done in the world's premier industrial fluidization laboratory—will encourage those of us who would like to see the two-phase flow idea applied more widely to fluidization processes.

A final point about Matsen's work is that—like so many good ideas—it was suggested almost simultaneously by someone else. Thus Matsen first presented his theory at the A.I.Ch.E. meeting in Los Angeles 1968, and the idea of using maximum bed height, though without an underlying theory, was presented independently by Hovmand (1968). The story has a happy ending; the two authors are to publish a joint paper and the marriage of true minds was contracted at the I.U.T.A.M. Symposium.

Section 5. Non-turbulent suspensions: motion of individual particles

The section on non-turbulent suspensions was introduced by Prof. S. G. Mason*. He restricted himself to neutrally buoyant particles, of a size large enough to be uninfluenced by Brownian movement, usually suspended in Newtonian fluid. He presented and interpreted a number of coloured cinéphotographic observations, of a quality that surpassed any seen by most of the audience. (Some could not believe that the movies were not animations!) For simple shear flow and low Reynolds number (based on the particle mean diameter and the local shear rate), the Jeffery orbits of rigid spheroids in simple shear flow are exactly verified. As the Reynolds number increases, the non-linear effects are observed to lead to orbits of maximum dissipation; with certain elastic or pseudoplastic fluids, however, a drift into orbits of minimum dissipation is observed—it was not made clear whether the drift using elastic fluids was the result of non-linear elastic forces alone. With deformable drops, a surface tension effect is introduced and the pattern of deformation and finally break-up was beautifully demonstrated. Reminding his audience that his work had been initiated by a study of wood-pulp fibres in paper making, Mason turned to the behaviour of long thin fibres; he referred to a simple theory based solely on the viscous compressive or tensile stresses induced in the straight fibre by simple shear motion in the fluid. An Euler buckling theory was shown to lead to sensible predictions for the onset of bending of the fibre; a maximum tensile stress condition equally gave good predictions for rupture of weak fibres, such as macro-molecules, e.g. DNA. Next he moved to electrical effects, which could produce a linear array of touching particles; at low shear rates, these rotate as rigid rods. (This same effect could be obtained by using a third liquid phase. Although outside the stated topics of the symposium, many of the most interesting effects were achieved by considering three liquid phases, of which one was continuous and the other two dispersed together in several modes depending on the various surface tensions.)

The second and most important part of the talk concerned two- or many-body interactions. For low Reynolds number, as theory predicts, these are shown to be symmetrical and reversible. Observation also confirmed the theory based on linear collisions for: (i) two-body collision frequency, according to the Smoluchowski equation; (ii) mean free path; (iii) statistics of lateral dispersion; (iv) angular velocity of collision doublets; (v) distribution of doublet lives; (vi) steady-state concentration of doublets; (vii) steady-state concentration of triplets; (viii) rotation of non-separating doublets. The role of triplet interactions in setting up permanent doublets was demonstrated; these have a slight offset in the z-co-ordinate direction when the shear flow is given by $u_y = Gx$. He noted that reversibility is lost when using liquid droplets.

Some significant results for transient effects in sheared dilute suspensions of rigid rods were reported. Despite reversibility, there appears to be a tendency for all initial distributions to approach, because of particle–particle interactions, a certain steady-state distribution which is anisotropic. By starting with all rods aligned perpendicular to stream lines a temporal damped fluctuation in mean properties is noted. To each instantaneous ensemble configuration corresponds a certain intrinsic viscosity (or, more precisely, a certain instantaneous rate of dissipation of energy—it is not possible in general to represent the stress system in a suspension of non-isotropic structure in terms of an effective viscosity, as was emphasized in a later talk by Batchelor*). The characteristic time for ensemble changes seemed to be of the same order as the rotation time for individual particles.

Finally Mason made brief reference to migration in Poiseuille flow. With Newtonian fluids, a tubular pinch effect, noted by Ségré & Silberberg, arises for solid spheres, which is an inertial (Reynolds number dependent) effect; deformable particles move towards the axis, independent of inertial forces. Using pseudoplastic fluids, solid particles seem to move toward the axis, but with elasticoviscous fluids, they move towards the wall.

The only other talk restricted to viscous effects acting on what might be called the colloidal scale was by Prof. J. D. Goddard*. He developed a formal theory of flow past deformable ellipsoids where thermal fluctuations (Brownian motion) are relevant; some simple solutions of the resulting diffusion-type equation were given for small effects of deformation and diffusion; these were expressed in continuum-mechanical form.

Several other speakers discussed relative-motion effects. Prof. B. Otterman* discussed the very small migratory forces and velocities that arise when spherical particles interact individually with fluid in a laminar mixing layer. They consider a situation in which the three relevant Reynolds numbers, R_p based on the relative translation motion $(\mathbf{u}-\mathbf{u}_p)$ between particle and fluid, R_k based on the local fluid shear rate, and R_Ω based on the particle's rate of rotation, are all $\ll 1$, with $R_k R_\Omega \gg R_p^2$. Using a formula of Saffman that predicts a transverse force of higher order than the drag force leading to relaxation of the relative translation motion, they adopt a perturbation technique to calculate migratory characteristics. These are said to agree with hitherto unexplained experimental observations. The important physical force is that proportional to

$$\operatorname{curl} \mathbf{u} \wedge (\mathbf{u}-\mathbf{u}_p)/|\operatorname{curl} \mathbf{u}|^{\frac{1}{2}},$$

i.e. caused by the interaction of shear and relative motion. Dr F. C. McMichael* presented a rather technical and interesting account of sedimentation devices in inclined tubes giving some formulae obtained quite simply (but not previously quoted) on the basis of pure gravity settling, but noting that the sediment on inclined tubes itself falls as a turbid layer down the slope. These dense turbidity currents can interact with the dilute settling taking place. This work and that of de Koning mentioned earlier were excellent examples of how a careful scientific approach to practical problems can suggest links with other physical phenomena and stimulate theoretical research.

Prof. J. O. Hinze* applied himself to the problem of where the 'virtual added mass' term should appear in the equations for a two-phase, solid particle and inviscid liquid, system. He took for simplicity a single sphere in an infinite fluid, and presented the momentum balance for the case of uniform acceleration of the fluid at infinity; dp/dx, the pressure gradient, is given by dU/dt where U is the velocity of fluid at infinity; the solitary spherical particle is itself moving relative to the fluid, for convenience in the same direction as the fluid, and is being accelerated. Hinze showed that dU_p/dt (U_p being the particle velocity) is related to dU/dt by an equation containing the added mass term. He then argues that the added mass term will not appear in the momentum balance equation for the fluid alone, and that for non-interacting particles, this result is again recovered. This would appear an obvious consequence of the definition of dp/dx

in terms of dU/dt alone and not therefore to resolve the question of what meaning is to be attached to the pressure p within a concentrated suspension: the experimenter naturally likes to relate p to a measurable stress.

Dr J. L. Eichhorn* discussed part of an obviously probing and sustained experimental investigation into the nature of the motions of fluidized particles. Using a turbulent, upwards-flowing airstream in a vertical circular pipe, one, two or more equal spheres were supported in dynamical (statistically steady) equilibrium. Régimes were delineated for a single particle: (1) rotating and vibrating about a horizontal axis with mean position near the centre of the tube; (2) rotating and vibrating about a vertical axis near the tube centre; (3) rotating near the wall and vibrating up and down. Stable arrangements for two to seven vibrating and rotating particles in a horizontal plane were demonstrated by ciné-photography. The corresponding vibrations in multiple layers of equal horizontal distribution led to vertical columns of spheres moving up-and-down more or less in concert. Data were given of the mean drag coefficient as a function of number and distribution of spheres, sphere-to-tube diameter ratio, density ratio, Reynolds and Froude number of the flow. The speaker restricted himself to observations, though clearly these could lead to considerable insight into mechanisms for fluidization and should form a suitable field for theoretical investigation.

Dr P. P. Koryavov* described purely numerical calculations for the flow of viscous fluids past one or two axisymmetric particles placed symmetrically on the axis of a circular tube. By evaluating the drag on the particles at different separations, steady aggregated flow velocities were deduced. No new techniques were involved; no surprising conclusions ensued. Dr Y. P. Gupalo* reported very briefly on a patented screw-extruder device for the separation of particles by density; the Archimidean screw motion tends to push particles in one direction, while the forced flow of liquid tends to convect them in the opposite direction. A simple theory based on plug flow for the fluid, which is turbulent, and Stokes resistance to sedimentation for the particles predicts a great improvement (on conventional methods) in the sharpness of separation for particles of different density and size, i.e. over quite a large range of mean diameter the critical density for neither forward nor backward motion in the screw is within an acceptably narrow range. This is observed in practice.

Prof. B. Gal-Or* used a cell model to predict the terminal velocity of an ensemble of falling drops, bubbles or solid particles, with various interfacial effects. The $\frac{1}{3}$-power dependence on concentration so obtained was criticized as being incompatible with a result of Burgers, which shows it to be linear, but this contradiction was partially resolved by the latter result being restricted to very low concentrations; the former is clearly empirically justified for higher concentrations—some even claiming it to be valid for all. Dr J. Litwiniszyn* described equations said to be useful in describing the colmatage-scouring process in the flow of suspensions through porous beds. These proved to be very similar to those describing the process of gas chromatography and was mathematically equivalent to those solved by chemical engineers in the case of flow with diffusion and reaction.

Section 6. Rheology of suspensions

Prof. J. G. Oldroyd* surveyed the field with an accent on the historical development of the subject. In common with Mason*, he chose systems with negligible relative motion, neglected diffusive effects and compressibility, and emphasized the role of the geometrical configuration and distribution of the particles. He noted that viscosity of the continuous liquid phases confers viscosity on the mixture, while elasticity in the solid dispersed phases introduces elastico-viscosity to the mixture. His theme was the derivation of rheological equations of state relating the stress history of an element to its deformation and temperature histories, using whatever constitutive parameters are relevant to the mixture in question. He emphasized the significance of the mathematical connexion between the behaviour of a real two-phase mixture and that predicted for an equivalent idealized homogeneous medium (a model). A mixture is equivalent to such a model medium if a finite confined volume of an infinite quantity of the model material can be replaced by the mixture and the result remain indistinguishable from the original as far as stress/deformation behaviour is concerned at large distances from the volume so replaced. [The importance of this concept seems to be that this definition avoids discussion of the meaning of stress and deformation within the mixture itself; it presupposes that no meaningful rheological concepts can be interpreted on any scale comparable with that of the dispersed phase.] He noted that this approach leaves the question of boundary conditions at a solid wall as a separate issue [a matter seldom appreciated by solvers of boundary-value problems!]. The characteristics of the equivalent continuum obtained by this definition are highly dependent on the orientation and distribution of particles involved: this he termed problem I and was still essentially unsolved, even for very restricted cases. The problem that had been successfully solved for many cases was problem II, obtained by embedding a unit particle at the centre of a sphere of liquid, which is itself embedded in the model continuum; the ratio of volumes of particle and liquid gives the concentration. [Clearly this special approach can apply only to very dilute mixtures and is the basis of many theoretical derivations beginning with that of Einstein.]

Oldroyd noted first those linear viscous analyses, such as those of Einstein, Guth & Simha and Kynch, carried out for suspensions of uniform spheres that are isotropic and can be interpreted in terms of apparent (or intrinsic) viscosities. The bulk of the talk was devoted to linear elastico-viscous theories, beginning with that of Fröhlich & Sack, followed by those of Oldroyd himself and culminating in those of Roscoe (1967) and Goddard & Miller (1967). The approach was to start from the particular, i.e. a Newtonian fluid of viscosity η containing particles with rigidity modulus μ, subject to small deformations, and to derive the relevant equation of state for the equivalent continuum. The first generalization is to use visco-elastic particles in an elastico-viscous liquid. The next generalization is to introduce large deformation of the mixture; this, as is now well known, leads to non-linear terms and the Jaumann (or other similar) derivatives. The problem of high particle concentration was not discussed. The consequences of this equation of state for mixtures subject to simple shear was explained, and reference was made to a little-known paper of Giesekus (1962), who calculated the differences of normal stresses to be expected for suspensions of rigid anisotropic particles.

[The relevance of these theories to an explanation of the rheological behaviour of polymer solutions was not touched upon at the Symposium. It is worth remarking, however, that a large amount of observational data has now been accumulated (see, for example, the reviews by Berry & Fox (1968) and Shen, Hall & de Wames (1968)) and interpreted in molecular terms, where pseudo-fluid mechanical arguments have been used to describe the interaction effects between coiled polymer molecules and the solvent fluid. Most of the latter work has been restricted to simple shear flow—by no means simple as far as the behaviour of polymer molecules is concerned—and so their extension to more general patterns of deformation will need the tensor formulation of continuum mechanics.]

Prof. G. K. Batchelor* considered the stress in a suspension of non-spherical rigid particles aligned parallel to each other, as they would be in a pure straining motion after some time. The deviatoric stress system that is generated in a dilute suspension produces a tension of order $\mu ena^3/(\log a/b)$ for spheroidal particles with principal diameters a and b when $a/b \gg 1$, where μ is fluid viscosity, n the particle number density and e the rate of extension parallel to the length of the particles. The interesting point about this result is that it yields a much greater extensional viscosity than the intrinsic viscosity noted in simple shear flow; in other words the elementary ideas of increased Newtonian viscosity just do not apply to suspensions in which the particles are oriented preferentially. This result, although not new, is often overlooked, and may be of importance in turbulent motions, particularly in connexion with drag-reduction. It was shown that a result of the same form holds for a dilute suspension of any slender bodies. A further calculation for concentrated suspensions of parallel slender particles, in which the flow is dominated by the parallel sliding motions taking place between particles which are close together laterally, shows that the tension is now of order $\mu ena^3/\log(1/2nab^2)$ where a and b are effective length and breadth. This result can be matched smoothly to that for a dilute suspension.

Dr W. F. Hall* introduced magnetic forces into the problem of the viscosity of suspensions of spheres. In colloidal particles (55–120 Å) which behave as fixed dipoles, sheared in a uniform magnetic field aligned either parallel or perpendicular to the streamlines, a viscosity increment due almost completely to magnetic resistance to rotation is observed. A calculation was presented for the case of non-interacting particles not subject to Brownian motion in steady-state motion, and compared with observations in an Ostwald viscometer. The comparison showed moderate agreement. Later discussion suggested that the aligned form for dipoles would be stable.

Dr V. N. Nikolaevsky* chose to describe dilute suspensions of solid rotating particles in terms of a continuum model exhibiting couple stresses. Much of the formal analysis seemed similar to that given by other authors. The existence of a couple stress in turbulent flow was discussed very briefly and appeared to be dependent on five phenomenological constants.

Prof. S. P. Sutera* reported on experimental observations of pressure drop and average tube concentrations for tube flow of suspensions of neutrally buoyant spheres and disks. He interpreted his results in terms of 3 régimes: (i) a tubular pinch effect (mentioned earlier) for dilute suspensions of small particles; (ii) partial plug flow for concentrated suspensions of medium-sized particles; (iii) complete plug flow, with wall exclusion effect, for very concentrated suspensions of large particles. The range of suspensions studied was covered by relative volume concentrations from 15 to 40 %, by tube-to-particle ratios between 5 and 114 and tube flow Reynolds numbers from 0·4 to 50. [In discussion, the possible role of the entry flow in changing particle concentrations was mentioned. It was not clear whether output concentrations were always equal to reservoir concentrations; this seemed important because the distribution of particles within the tube seemed to be inferred from the ratio of tube-to-reservoir concentrations.]

Prof. R. B. Krone* presented results on the rheological behaviour of aggregating cohesive sediments. He interpreted his results, successfully for his purposes, in terms of a Bingham plastic behaviour represented for simple shear by

$$\tau = \tau_B + \eta_d \frac{du}{dz}; \quad \eta_d = \eta_l e^{2 \cdot 5\phi},$$

where η_d is the overall viscosity of the mixture containing a concentration ϕ of aggregated material in a liquid of viscosity η. This exponential law was perhaps unwisely supported by theoretical arguments which were strongly attacked by several questioners, although it appeared to correspond to observations. Photographs showed that the speaker was dealing with very complex assemblages of particles, and it was encouraging that he had been able to make any progress whatever. His systems showed strong evidence of hysteresis (of pseudoplastic nature) under varying rates of shear.

Dr J. J. Benbow* explained briefly how the extrusion of clay/water/bentonite mixtures from screw-extruders through dies could be interpreted in terms of a plastic material model with Coulomb frictional boundary conditions at metal interfaces. Although rather outside the main topics discussed at the conference, his approach could be relevant to other situations. He had investigated the effect of taking other boundary conditions such as hydrodynamic lubrication, giving velocity-dependent boundary stress, but this proved not to fit the observations.

The final two talks by Prof. P. Naghdi* and Prof. R. M. Bowen* were couched in formal continuum-mechanical form. The first considered a theory for relative flow of distinct interpenetrating continua based on the energy equation and on an entropy production inequality. The second, although admirably presented, seemed only remotely connected with the main themes of the conference, in that virtually no dynamical interaction between the two phases was considered. Both talks provided conceptual frameworks rather than predictive or applicable results. They did include the possibility of inertial and gravitational effects but no attempt was made to calculate or specify at all closely the nature of the interaction term between the phases.

Looking back over the rheological treatments described, it appears that there are still problems concerned with the behaviour of even limitingly dilute suspensions, certainly as regards their behaviour in general uniform deformations imposed on a gross scale. The outstanding problem is that of the particle–particle interactions which occur when the concentration of the suspension rises; as yet we have inadequate means, either analytic or experimental, to investigate even the gross uniform behaviour of such suspensions.

For many practical purposes, such knowledge of uniform systems may not be all we need, because non-uniformities in the flow field may occur on such a scale that they interact with the rheology of the medium. Diffusion (Brownian or other) of particles may occur and so concentration becomes a field variable. The question then arises whether this concentration variation itself eliminates the applicability of the continuum concept, and even whether in large scale, or very intense, motions a statistical unsteadiness of a turbulent nature can arise to confer wholly unexpected behaviour on the mixture. To take a simple example, soils can exhibit plastic behaviour with well-defined slip surfaces; as they become wetter, they continue to behave plastically until the water becomes a continuous phase and they behave as muds: what happens at the critical volume ratio? Do local variations in concentration occur throughoutthe medium? —we certainly know that they do at the walls. These are not irrelevant considerations, as Massimilla's detailed photographs showed for fluidized beds. The problem is one of relating statistically unsteady phenomena to models expressed in terms of systems of differential equations involving only mean values of physical quantities. This problem has not been resolved for high Reynolds number turbulence, because of the broad spectral distribution of eddies. It becomes almost trivial in the case of 'laminar' flow of gases or liquids because of the very large difference in length scales between the discrete particles in the fluid (or their mean free paths) and the continuum length scale. The most intractable of the problems raised at the symposium seemed to involve effects where relevant length scales overlapped.

Chairmen's summaries

The symposium ended with short summaries presented by the Section Chairmen. Prof. F. Henderson felt that no substantial progress had been made in the field of suspended load; he thought that too much emphasis was placed on eddy diffusivity of particulate matter, ϵ_s, and that not enough experimental work had been done; he suggested that the energy loss due to the load lagging the fluid needed investigation. Prof. J. F. Kennedy's theme was the moving bed, whose dynamics could be described in terms of entrainment mechanisms and the stability of bed forms; these lead to the gross properties of alluvial channel flows. He emphasized the role of the turbulent velocity gradient and what is generally called upwelling in providing the necessary lift for suspending bed particles, though he noted also the effects due to mean velocity gradients and bed porosity. He predicted an important role for electronic instrumentation in the future. In some cases, temperature can be important, even though no mention had been made of it as a factor. He reviewed the uncertainties in stability theory connected with local sedimentation rates; clearly a phase shift is present but its origin is not clear. Although the hydraulic roughness of a flowing stream is still not understood, he looked forward to the time when empirical régime theory could be replaced by a more soundly based approach.

Dr H. K. Moffatt found it difficult to find a common thread linking the papers given in his section, though he paid tribute to the excellent survey given by Owen. Prof. R. Jackson reminded the audience that dynamics forms only one small part of the interest in fluidized beds, and that kinetics, heat and mass transfer are much more important from a practical point of view. He reminded the audience of the considerable success of the interpenetrating fluid continua theories that had been questioned by Rietema. Looking ahead, he felt that a continuum theory that took account of large relative velocity between the phases could be usefully applied to fluidization mechanics; the outstanding problem was the prediction of bubble sizes and their distribution.

Prof. H. Brenner noted a lack of interaction between the work presented in §§5 and 6; this is clearly the area in which research is most needed, and some of the problems are conceptual and not merely analytic. Although the basic problem can be stated, the relating of the behaviour of individual particles to that of gross multi-phase continua will need a statistical approach. Prof. R. S. Rivlin took up the question of the continuum approach: little progress had been made in reconciling the three approaches that have been employed, viz. (1) find out the experimental facts, using theory only to obtain meaningful results; (2) construct physical models [a mathematical not an engineering exercise] and proceed from the particular to the general; (3) produce a general theory within a broad framework. He commented on the strengths and weaknesses of each, and reminded the audience of the great importance of method (1), which provides our knowledge of real materials; method (3) is often attacked as being unrelated to real materials, but he suggested that it is often method (2) which misrepresents the facts.

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